Power Law Stellar Distributions

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(February 2, 2008)

Abstract

The density profiles and other quantities of physical interest for spherically symmetric systems are computed by assuming that a collisionless stellar gas may relax to the non-Gaussian power law distribution suggested by the nonextensive kinetic theory. There are two different classes of solutions. The first class behaves like a subset of the polytropic Lane-Emden spheres, whereas the second one corresponds to a transition between two different polytropic indices. Unlike the isothermal Maxwellian sphere, the total mass and sizes of both classes are finite for a large range of the nonextensive q-parameter.

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I. INTRODUCTION

It is widely assumed that self-gravitating stellar systems like globular clusters are completely or almost completely relaxed because they are changing very slowly, or equivalently, the characteristic evolution time scale is too long. In particular, this means that stellar encounters combined with the action of some long range kinetic processes like "phase mixing" and "violent relaxation" have driven their velocity distributions toward a collisionless kinetic steady state (Spitzer & Härm 1958, King 1962, Michie 1961, 1963, Lynden-Bell 1967, Spitzer 1987). Although considering the numerous and important contributions to this field, an overlook in the recent literature suggests that the equilibrium (or near-equilibrium) distribution of these relaxed systems is still not firmly established (beyond doubt), and basically remains as an open question (Drukier et al. 1992, Meylan & Heggie 1997, Katz & Okamoto 2000, Chavanis 2002).

The phase-space density for spherically symmetric systems was initially described by the equilibrium Maxwell-Boltzmann (MB) distribution

$$f(\mathbf{r}, v) = \rho_1 \left(2\pi\sigma^2\right)^{-3/2} \exp\left(-\frac{\frac{1}{2}v^2 + \Phi(\mathbf{r})}{\sigma^2}\right) \tag{1}$$

where Φ is the potential per unit mass, ρ_1 is the density in the absence of the force field and σ is the velocity dispersion. However, the computation of the associated mass density gave rise to a very serious problem, namely: the total mass is infinite (Chandrasekhar 1960, Binney & Tremaine 1994). This undesirable feature is more easily seen from the particular solution, $\rho(r) = \sigma^2/2\pi G r^2$ (singular isothermal sphere) which predicts an infinite total mass. Many ad hoc phenomenological distributions were proposed to solve this problem with basis on Jeans' theorem: any steady state solution of the collisionless Boltzmann (or Vlasov) equation depends on the phase space coordinates only through isolating integrals of motion (Jeans 1929).

A different route was initiated by Spitzer and Härm (1958), and further worked out by Michie (1961, 1963) and King (1962, 1965, 1966). It was recognized that star clusters

cannot be exactly described by the expected MB equilibrium distribution. However, it was also assumed that relaxation processes always drive the stellar distribution as far as it can to a quasi-MB final state which should be determined, for instance, by solving the non-equilibrium Fokker-Planck equation. This line of inquiry lead to some interesting and consistent results among them the lowered isothermal sphere, as well as the anisotropic models proposed by Michie.

In this article we consider a different approach. With basis on the q-power law equilibrium distribution we determine the radial and projected density profiles for two large classes of isothermal stellar systems. It should be recalled that the nonextensive treatment for a stellar collisionless systems was first considered by Plastino & Plastino (1993) through a variational principle where the Tsallis (1988) entropy formula was maximized taking into account the constraints imposed by the total mass and energy density. Here, we consider directly the nonextensive distribution which follows naturally from the kinetic equilibrium q-entropy formula (Lima et al. 2001). An attractive feature of this power law distribution is that the models are analytically tractable in such a way that a detailed comparison with the standard Maxwell-Boltzmann approach is immediate. As we shall see, there are at least two classes of solutions with finite mass and radius. Actually, the first class is not new, however we discuss some related subtleties not considered in the quoted articles.

The paper is structured as follows. Next section we set up the basic equations and discuss the density profiles to the first class of nonextensive spherically symmetric self-gravitating stellar systems based on Tsallis' distribution. Section 3 discuss the main features of the trunctated models, and, in section 4, we resume briefly the main results.

II. POWER LAW STELLAR SPHERES

Let us now consider the following non-Gaussian velocity distribution

$$f(v) = \frac{\rho_1 C_q}{(2\pi\sigma)^{3/2}} \left[1 - (1-q)\frac{v^2}{2\sigma^2}\right]^{\frac{1}{1-q}} \qquad (2)$$

where the q-parameter quantifies to what extent the distribution departs from the standard Gaussian form (Silva et al. 1998, Lima at al. 2001, Mendes & Tsallis 2001, Kaniadakis 2001). Actually, it is a power law if $q \neq 1$, whereas for $q = 1$ it reduces to the Maxwellian function. This result follows directly from the known identity $\lim_{\mathrm{d}\to 0}(1+\mathrm{dy})^{\frac{1}{\mathrm{d}}} = \exp(\mathrm{y})$ (Abramowitz & Stegun 1972). The quantity C_q is a q-dependent normalization constant given by (Lima et al. 2002)

$$C_q = (1-q)^{1/2}(\frac{5-3q}{2})(\frac{3-q}{2})\frac{\Gamma(\frac{1}{2}+\frac{1}{1-q})}{\Gamma(\frac{1}{1-q})} \quad for\, q \leq 1 \tag{3}$$

and

$$C_q = (q-1)^{3/2}\left(\frac{3q-1}{2}\right)\frac{\Gamma\left(\frac{1}{1-q}\right)}{\Gamma\left(\frac{1}{1-q}-\frac{3}{2}\right)} \quad for\, q \geq 1, \tag{4}$$

which reduce to the expected result in the limit $q = 1$. For instance, for $q \leq 1$ we define $z = (1-q)^{-1}$ so that $z \to \infty$ if q goes to unity. In addition, from the identity $\lim_{|\mathrm{z}|\to\infty}\mathrm{z}^{-\mathrm{a}}\Gamma(\mathrm{a}+\mathrm{z})/\Gamma(\mathrm{z}) = 1$ (Abramowitz & Stegun 1972), one may see that $C_1 = 1$, thereby showing that the normalization of the standard Maxwellian distribution is recovered in this limit.

The power law distribution (2) can be deduced from two simple requirements: (i) isotropy of the velocity space, and (ii) a suitable nonextensive generalization of the Maxwell factorizability condition, or equivalently, the assumption that in this enlarged framework $f(v) \neq f(v_x)f(v_y)f(v_z)$. It was also shown that for $q > 0$, the above distribution function satisfies a generalized H-theorem, and its reverse has also been proved, that is, the collisional nonextensive equilibrium is given by the Tsallis' power law velocity distribution (Lima et al. 2001). In the last few years, several applications of this equilibrium power law velocity distribution have been done in many disparate branches of physics (Liu et al. 1994, Bhogosian 1996, Lima et al. 2000, Tsallis et al. 2001). In the astrophysical context, Taruya and Sakagami (2002), investigated in detail the problem of instability (gravothermal catastrophe) working in the so-called (u,v) plane (Milne's variables). Further, they shown that the king model fails to match the simulated distribution function, especially at cut-off scales (Taruya & Sakagami 2003). Even the Jeans gravitational instability criterion for a

collisionless system was recently discussed in this enlarged framework (Lima et al 2002). In particular, for power law distribution with cut-off, it was shown that such a system presents instability even for wavelengths of the disturbance smaller than the standard critical Jeans value. For a selfgravitating system, the nonextensive phase-space density reads (Lima et al. 2002a)

$$f(\mathbf{r},v)=\frac{\rho_1 C_q}{(2\pi\sigma)^{3/2}}\left[1-(1-q)\left(\frac{\frac{1}{2}v^2+\phi(\mathbf{r})}{\sigma^2}\right)\right]^{1/(1-q)} \tag{5}$$

which can also be obtained by integrating the Vlasov equation. As should be expected, the above expression reduces to equation (1) in the limiting case $q=1$.

Let us now consider the velocity distribution (5) to build a set of stellar systems whose structure is sustained by their respective gravitational field. Following standard lines, the granularity of the star system is ignored, the gravitational potential is assumed to be a slowly varying function of position, and any change in the physical properties due to collisions or evolution of the stars are neglected. The radial dependence of the mass density is obtained by integrating (5) over all allowed velocities

$$\rho=\frac{4\pi\rho_1 C_q}{(2\pi\sigma)^{3/2}}\int\left[1-(1-q)(\frac{\frac{1}{2}v^2+\phi(\mathbf{r})}{\sigma^2})\right]^{1/(1-q)}v^2dv \tag{6}$$

whereas the gravitational potential ϕ must be determined from Poisson's equation

$$\frac{1}{r^2}\frac{d}{dr}(r^2\frac{d\phi}{dr})=4\pi G\rho. \tag{7}$$

In order to simplify further the integral (6), it is convenient to introduce the dimensionless energy per unit mass of a star

$$\varepsilon=\frac{\frac{1}{2}v^2+\phi}{\sigma^2} \tag{8}$$

and from the argument of the power law in (5), this quantity is restricted by $\varepsilon\leq\frac{1}{1-q}$, in order to guarantee a definite real valued distribution function (unless explicitly stated, in what follows we consider only the case $q\leq 1$). Now, inserting ε into (6) one has

$$\rho=\frac{4\pi\rho_1 C_q}{(2\pi\sigma)^{3/2}}\int_{\frac{\phi}{\sigma^2}}^{\frac{1}{1-q}}[1-(1-q)\varepsilon]^{\frac{1}{1-q}}(\varepsilon-\frac{\phi}{\sigma^2})^{\frac{1}{2}}d\varepsilon \tag{9}$$

and a simple integration furnishes

$$\rho = \rho_q[1 - (1-q)\frac{\phi}{\sigma^2}]^{\frac{5-3q}{2(1-q)}} \tag{10}$$

where the density scale is expressed in terms of the complete Beta function by $\rho_q = \rho_1(1-q)^{-\frac{3}{2}}B(\frac{3}{2},\frac{2-q}{1-q})$. In the limit $q = 1$ one finds $\rho_q = \rho_1$ with the power-law becoming the exponential, and as expected the standard Maxwellian result is readily recovered. Substituting (10) into Poisson's equation, and introducing the pair of dimensionless quantities defined by $x = r/r_o$ and $\theta = -\sigma^{-2}\phi(r)$, where $r_o = \sqrt{4}\pi G\rho/\sigma^2$, we find the Lane-Emden type equation

$$\frac{1}{x^2}\frac{d}{dx}(x^2\frac{d\theta}{dx}) = -[1 + (1-q)\theta]^{\frac{5-3q}{2(1-q)}} \tag{11}$$

whose importance for stellar dynamics is largely known. First, we notice that in the limit $q = 1$ the above equation reduces to

$$\frac{1}{x^2}\frac{d}{dx}(x^2\frac{d\theta}{dx}) = -e^{\theta} \tag{12}$$

which is exactly the Lane-Emden form for a Maxwellian isothermal sphere (Chandrasekhar 1960, Spitzer 1987, Binney & Tremaine 1994). The solution of this equation subjected to the boundary conditions $\theta = d\theta/dx = 0$ at $x = 0$ has been numerically computed by Chandrasekhar and coworkers (see Chandrasekhar 1960 and references there in). Some important theorems has been proven both for the Maxwellian isothermal sphere and the Lane-Emden equation. Such a study can also be extended to equation (11) adopting the same boundary conditions.

Now, it should be recalled that the gravitational potential is defined up to a constant value in an arbitrary level. Therefore, one may also choose ϕ_o in such a way that $\phi \rightarrow \phi - \phi_o$ subjected to the restriction that both the potential and the density goes to zero at infinite. With that choice, the arbitrary constant for $q \neq 1$ is $\phi_o = 1/(1-q)$ and from (10) we obtain the following expression

$$\rho = \rho_q\theta^{\frac{5-3q}{2(1-q)}} \tag{13}$$

whereas the differential equation (11) becomes

$$\frac{1}{x^2}\frac{d}{dx}(x^2\frac{d\theta}{dx}) = -\theta^{\frac{5-3q}{2(1-q)}} \tag{14}$$

which is the canonical form of a polytropic Lane-Emden equation of index

$$n = \frac{5-3q}{2(1-q)}. \tag{15}$$

In figure 1 we display the profile ρ/ρ_q for some values of the q parameter. The labels under each curve correspond to the value $10q$ and since the density profiles are identical to the classical polytropic solution we obtain the known restriction that the total mass diverges above the polytropic index $n = 5$ since the Lane-Emden spheres have finite mass only for $n \leq 5$. In terms of the q-parameter this condition is simply translated as $q \leq 5/7$. On the other hand, from the second law of thermodynamics we know that $q \geq 0$ (Lima et al 2001). Therefore, models with finite mass and sizes are obtained if the nonextensive parameters lies on the interval $0 \leq q < 5/7$. Such a limit was previously obtained by Plastino and Plastino (1993) maximizing Tsallis entropy. The unique difference is that it was referred to as 9/7 because the distribution with cut-off was written as q-1 instead of 1-q. In fact, substituting $q \rightarrow 2-q$ we see that 5/7 goes to 9/7, with the latter becoming a lower bound as should be expected. Note that the physical lower bound constraint ($q \geq 0$) means that $n \geq 5/2$. Therefore, Tsallis distributions whose total energy has a cut-off ranges only a half of all possible Lane-Emden distributions with finite mass. However, since the above Lane-Emden type equations are still valid for $q > 1$, one may see that $n \geq 0$ means $q \geq 5/3$, and in the limit $q \rightarrow \infty$ the index $n \rightarrow 3/2$. Hence, we conclude that Tsallis power law statistics do not describe the class of polytropic distributions contained on the range $3/2 < n < 5/2$ since this range requires negative values of q. These results can be seen directly in the plane (n,q) represented in figure 2.

At light of such results, one may be tempted to conclude that a subset of Lane-Emden stellar polytropes can be regarded to as nonextensive collisionless isothermal spheres, thereby giving a kinetic justification to this class of stellar configurations. It is argued here, however,

that such an identification is misleading since polytropic models are defined by one of their equation of state, namely: $p\rho^{-\gamma} = const$, or equivalently, $T\rho^{1-\gamma} = const$, where $\gamma = 1+1/n$ is the polytropic index. Hence, the temperature in polytropic models depends on the position, whereas the class of nonextensive spheres (including the Maxwellian case) has constant temperature which can formally be defined by the velocity dispersion $\sigma = \sqrt{k_B T/m}$.

In figure 3 we display the projected mass density Σ for the same set of the q-parameter illustrated in figure 1. These profiles illustrates the surface density profile that should be obtained in an astronomical object described by this class of Tsallis models.

It is worth notice that Tsallis isothermal spheres have a fraction of the stars with positive energy and even so they are bounded to the main structure whose total mass is finite for $q < 5/7$. In this concern, one may ask what happens if the energy is constrained by $\varepsilon \leq 0$ as usually adopted for bounded structures. Physically, one would expect finite structures for a larger range of the q-parameter. This problem it will be discussed next section.

III. TRUNCATED NONEXTENSIVE STELLAR MODELS

Let us now consider a class of nonextensive stellar models which is obtained by assuming that only objects with $\varepsilon \leq 0$ are present in the distribution. It can be viewed as a simple generalization of the truncated model proposed by Wooley (1954). The corresponding density profile as a function of the gravitational potential assumes the form (Cf. equation (9))

$$\rho = \frac{4\pi\rho_1 C_q}{(2\pi\sigma)^{3/2}} \int_{\frac{\phi}{\sigma}^2}^{0} [1-(1-q)\varepsilon]^{\frac{1}{1-q}} (\varepsilon - \frac{\phi}{\sigma^2})^{\frac{1}{2}} d\varepsilon \tag{16}$$

where integration limits are defined by equation (8). As one may check, this integral can be expressed in terms of hypergeometric functions in the form

$$\rho = \frac{C_q(\eta(\varphi))^{\frac{1}{1-q}}}{\eta_*^{\frac{3}{2}}} \frac{2}{3} (-\frac{\varphi}{\eta_*})^{\frac{3}{2}} F(\frac{3}{2}, -\frac{1}{1-q}, \frac{5}{2}, -\frac{\varphi}{\eta_*}) \tag{17}$$

where $\eta(\varphi) = 1-(1-q)\varphi$, $\eta_* = \frac{1-(1-q)\varphi}{(1-q)}$, and $\varphi = \frac{\phi}{\sigma^2}$. In order to work out a numerical solution it is more convenient to express this density profile in a form where the integration

limits are fixed as

$$\rho = C_q(-\varphi)^{3/2}\int_0^1 [1-(1-q)\varphi\chi]^{\frac{1}{1-q}}(1-\chi)^{\frac{1}{2}}d\chi \tag{18}$$

and one may verify that at the outer halo region, where $|\varphi| << 1$, the density profile behaves like

$$\rho \simeq \rho_q(-\varphi)^{3/2} \qquad\qquad |\varphi| << 1 \tag{19}$$

corresponding to a polytropic spherical profile having Lane-Emden index $n = 3/2$. In the central region, where we might adopt the approximation $|\varphi| >> 1$, the density profile reduces to the expression

$$\rho = \rho_q(-\varphi)^{\frac{5-3q}{2(1-q)}} \tag{20}$$

which corresponds to a Lane-Emden index $n = (5-3q)/2(1-q)$. Therefore, if all the stars has velocity smaller than the escape velocity, the general density profile in terms of the gravitational potential can be described as a smooth transition between two different polytropic spheres, where the case $n = 3/2$ behaves like an attractor for the outer halo region. This behaviour is illustrated in figure 4. As a further consequence all these truncated models have finite mass extent due to the very steep density profile dominating the external region.

The density profile of these truncated models can be easily obtained by a numerical integration of the the Poisson equation subject to the auxiliary condition given by equation 17. The integration begins at $r = 0$ by specifying a value φ_0 for the gravitational potential at the center of the mass distribution of the model. Knowing the potential we estimate the central density predicted by equation 17. Due to the continuity equation the density gradient at the center is null, and the same happens with the gradient of the gravitational potential. Once the density and the gradient of the potential are determined we apply a discretization algorithm to the second order Poisson equation for a grid of radial points. The algebraic approximation is used to predict the gravitational potential, and its gradient, in the next point of the grid. The whole process is iterated by solving the density in the next

point of the grid and applying recursively the discretization of the Poisson equation. The overall numerical integration stop at the point where the gravitational potential is null. At that point the corresponding density is also null and we reach the surface of the truncated model.

In figure 5 we present the result of applying this integration algorithm for the cases $q = 1$, 0.9 and 0.8. In each panel the continuous line corresponds to the non truncated model. This model corresponds to the asymptotic case when the central gravitational potential is sufficiently high to sample the polytropic structure. In the particular case of $q = 1$ we recover the classical isothermal sphere. On the other extreme we have the situation where the central gravitational potential tends to zero. In that case we can see from figure 4 that the structure is described by a $n = 3/2$ politrope independently from the q value. For that reason the density profiles for the three models in figure 5 are exactly the same when $\varphi_0 \to 0$. For intermediate values of the central potential the models tend to be quite different depending on the exact value of q. In that figure we have chosen in each panel two values of φ_0 corresponding to intermediate density profiles.

The density profiles can be integrated along the line of sight so that we can estimate the projected mass density. Assuming that the light trace the mass and that the mass to luminosity ratio is constant we can scale this to the surface brightness. In figure 6 we present this surface brightness as a function of the $r^{1/4}$ radial coordinate. The symbols are the same as in the previous figure. Again we can see that large values of φ_0 tend to be closer to the nontruncated polytropic models as indicated by the continuous line. When $\varphi_0 \to 0$ the models are described by the surface mass density obtained from the $n = 3/2$ polytrope.

IV. FINAL COMMENTS

Two simple applications of Tsallis' power law kinetic distribution have been discussed. The proposed kinetic models represent the mass distribution of collisionless gravitational stellar systems. The use of the Tsallis distribution introduces an extra parameter measuring

to what extent the velocity distribution departs from the standard Maxwell-Boltzmann law which is normally used to represent these objects. It has been proposed as a viable and well grounded alternative to the standard equilibrium approach in the presence of long range forces as happens in the astrophysical context.

As we have seen, the Poisson equation combined with Tsallis distribution usually reproduces spherically symmetric structures resembling the classical polytropic spheres. If the natural cut-off is imposed ($q < 1$), the polytropic index n is closely related to this Tsallis parameter. The basic result is that models with $q < 5/7$ have finite mass while above this limit the mass is divergent. The radial and projected density profiles were obtained by solving numerically the Poisson equation for a large range of the nonextensive parameter. Moreover, if objects with positive energies are excluded, one can build a larger set of truncated models with finite mass and sizes. For this class of models the corresponding profiles were also numerically determined (see figures 5 and 6). For both cases, the predicted profiles should be compared with that ones observed for globular clusters. In particular, since energy truncated models are presumed to describe the relaxed state of clusters, it is interesting to investigate their connection with possible nonextensive extensions of the spherically symmetric Michie-King models. Finally, we also remark that the possibility to include elliptical galaxies in this generalized framework cannot be discarded. These issues will be discussed in a forthcoming communication.

Acknowledgements: The authors are grateful to Raimundo Silva and Janilo Santos for helpful discussions. This work was partially supported by Pronex/FINEP (No. 41.96.0908.00) and CNPq (62.0053/01-1-PADCT III/Milenio).

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FIGURES

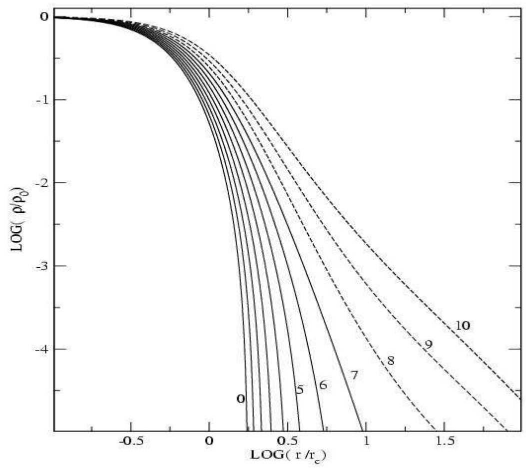

FIG. 1. Spatial density profile for different values of q. For $q < 5/7$ all profiles have finite extent and finite mass, while for $5/7 < q < 1$ the models extend to infinite and the total mass diverges. The limiting value $q = 5/7$ corresponds to $n = 5$ polytropic model.

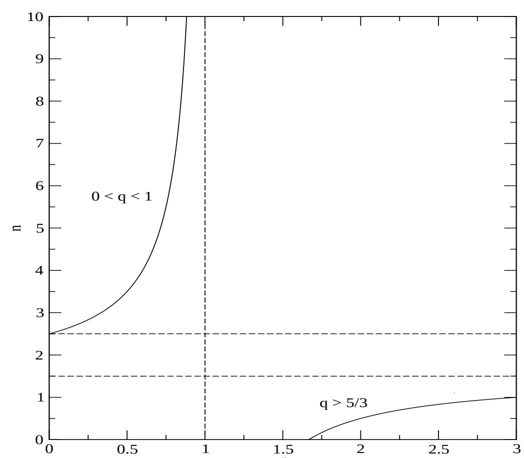

FIG. 2. The q index of the Tsallis isothermal description versus the polytropic index n. In the left region we have $0 \leq q < 1$ and the corresponding polytropic models are restricted to the interval $5/3 \leq n < \infty$. The polytropic models $0 \leq n \leq 3/2$ are described by Tsallis models having $q > 5/3$. Note that polytropic models defined by $3/2 < n < 5/2$ do not have a corresponding Tsallis model.

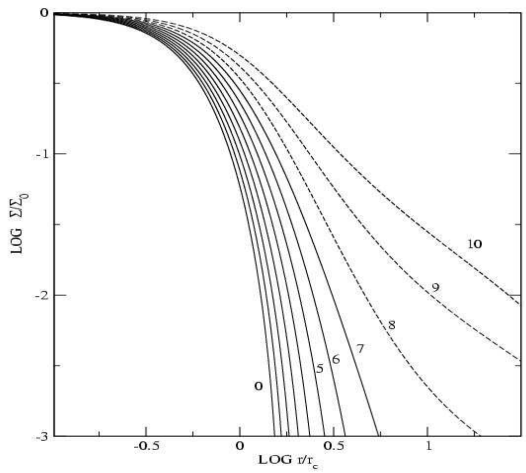

FIG. 3. Projected mass distribution for different values of q, obtained by numerical integration of the profiles presented in figure 1.

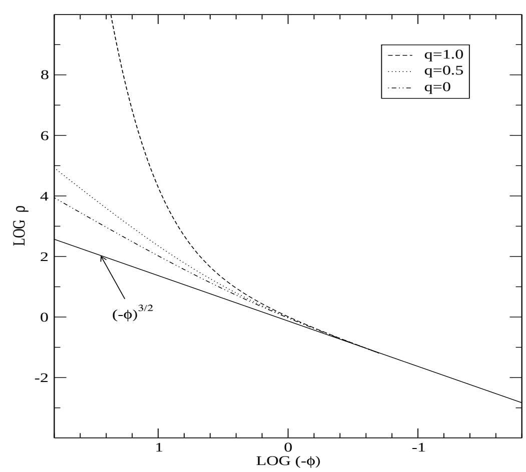

FIG. 4. A class of truncated models can be obtained by extracting from the density distribution all objects with positive energy. In that case the density profile changes and can be described by a superposition of two limiting polytropic models.

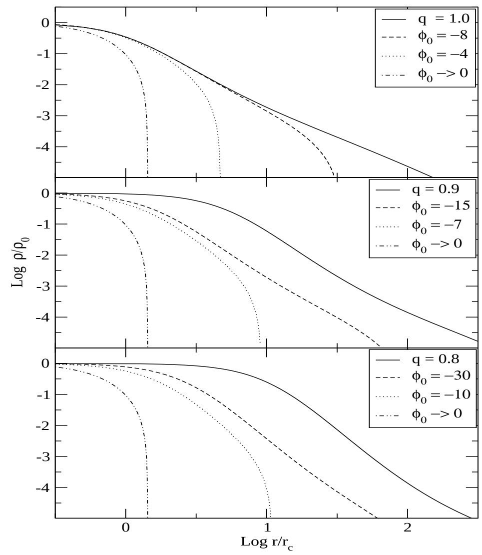

FIG. 5. Density profiles predicted for the truncated model corresponding to $q = 1$, 0.9 and 0.8. The continuous lines in each panel describes the nontruncated polytrope corresponding to the limiting case of large φ_0. When $\varphi_0 \to 0$ the models approximate the limiting $n = 3/2$ polytrope independently of the value of q.

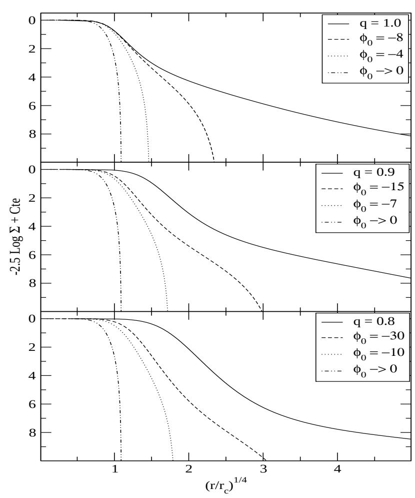

FIG. 6. Projected mass density for each of the density profiles of figure 5.